## Lab 1: Measurements and Error Propagation

## Equipment

- Computer, Calculator
- Ruler
- String
- Any tin can (a relatively large can of either spaghetti sauce, or beans, or tuna)

The only quantities in nature that are exact are integers ( $1,2,3, \ldots$ ). For example, the number of people in a room is an exact whole number (hopefully!). Any quantity that can take on an inbetween value could have any real number value. Unfortunately, we can only write decimal numbers with a limited number of decimal places. That means that if we are concerned with the length of an object, we can only record that length with a limited resolution. Scale of any given instrument is limited by the value of the smallest increment of that scale. The uncertainty due to resolution is half of the smallest graduation of the instrument.

Of course, there are other reasons that we may not know the exact value of a quantity. We may have a limited ability to measure the quantity. If we measure the height of a person with a measuring tape, for example, it may be difficult to make sure that we line up the ends of the measuring tape with the top and bottom of the person, that they stand up straight, that their hair isn't affecting the measurement, etc. Repeated measurements will involve these factors in different ways, leading to different results. That means that the process of measuring has limited accuracy. There no such thing a perfectly accurate method of measurement. One way of determining the uncertainty due to accuracy is to repeat a measurement many times and use the standard deviation of the measurement.

Any difference between a written value and the actual real-world value is an error. Generally, we do not know the error. But through experience we can put a limit on how big that error might be. This limit is called the measurement uncertainty in the value. Usually, the measurement uncertainty is determined based on the largest contributions. For example, when a person is using a stopwatch, the uncertainty may come from two sources. The stopwatch has an increment of 0.01 s , contributing an uncertainty of 0.005 s . But the person's reaction time could be 0.2 s . In this case, the reaction time overwhelms the stopwatch uncertainty. So, we estimate the uncertainty of "a person using a stopwatch" to be 0.2 s .

This lab will determine the density of a canned food along with the uncertainty of the obtained value. The definition of density is:

$$
\text { Density }=\frac{\text { Mass }}{\text { Volume }}
$$

To determine experimentally the density of an unknown material, we need two values: the mass and volume of the material. Each measurement comes with some uncertainty. Those uncertainties
will propagate in the calculation of the density. To evaluate the calculated values, we will compare them to either accepted values or values listed on the internet.

## Part 1: Calculating the value of $\pi$.

The circumference of a circle is related to the diameter of a circle:

$$
\begin{equation*}
C=\pi \cdot d \tag{1}
\end{equation*}
$$

Here, $\pi=3.14159 \ldots$ is an accepted value of a well-known irrational number.
The value of $\pi$ could be calculated from measured values of circumference and diameter while using formula (1). To assess the accuracy of the experimentally determined value of $\pi$, it must be compared with the accepted value. The result of the comparison or \% Error is the absolute value of the difference between two values divided by the accepted value, and it is presented in a percentage format.

1. Use a metric ruler to measure the circumference of a tin can. To measure the curved distance with a straight ruler, use a piece of string to wrap it around the can.
2. Using a ruler, measure the diameter of the can. Place the ruler across the bottom of the can while holding the zero mark firmly on the edge of the can. Move non-zero end of the ruler around opposite edge till the ruler reads the largest number.
3. Record the measured values along with estimated uncertainties, $\Delta \mathrm{C}$ for circumference and $\Delta \mathrm{d}$ for diameter. There always will be an instrumental uncertainty that is a half of the smallest increment of the ruler. Methodological uncertainty should be estimated based of the way the measuring was done. Example: how precisely was the sting cut? Was your thumb contributing to the uncertainty?
4. Use formula (1) and the measured values to calculate the value of $\pi$.
5. Compare your value of $\pi$ to the accepted value by calculating $\%$ Error.
6. Propagated the uncertainties in measured values into a calculated value of $\pi$ by computing the percent uncertainty, $\left(\frac{\Delta C}{C}+\frac{\Delta d}{d}\right) 100 \%$.

| Description | Value | Uncertainty |  |  |
| :---: | :---: | :---: | :---: | :---: |
| Circumference (mm) |  |  |  |  |
| Diameter (mm) |  |  |  |  |
| Experimental Value of $\pi$ |  | 3.14159 |  |  |
| Accepted Value of $\pi$ |  |  |  |  |
| $\%$ Error |  |  |  |  |

Table 1: Quantities measured and/or calculated in Part 1. Re-write the caption for the report such that it has complete information about the values from the table including description of how they were obtained (if measured, by what instrument? if calculated, by what formula?)

## Part 2: Calculating the Density.

The volume of a cylinder can be determined by using the formula

$$
\begin{equation*}
V=A h=\pi r^{2} h \tag{2}
\end{equation*}
$$

$r$ is the radius, and $h$ is the height of the cylinder. Since a tin can is essentially a cylinder, its volume could be calculated using formula (2).
The density of a substance can be determined by using the formula

$$
\begin{equation*}
\text { Density }=\frac{\text { Mass }}{\text { Volume }} \tag{3}
\end{equation*}
$$

1. Using a ruler, measure the height of the tin can and calculate the radius of the can from the value of the diameter measured in part 1 .
2. Record the measured values along with estimated uncertainties, see part 1.
3. Using formula (2), calculate the volume of the tin can. Use the value of $\pi$ obtained in part 1 .
4. Propagated the uncertainties in measured values into a calculated value of volume by computing the percent uncertainty, $(\Delta \pi / \pi+\Delta \mathrm{h} / \mathrm{h}+\Delta \mathrm{d} / \mathrm{d}+\Delta \mathrm{d} / \mathrm{d}) 100 \%$.
5. Using the mass of the product listed on the can (in grams) and formula (3), calculate the density of the product.
6. Search the actual density of the product ( you may use https://www.aqua-calc.com/ or the site of the company that manufactured the product).
7. Compare the value of density computed by you to the value found on the net by calculating $\%$ Difference. \%Difference is the absolute value of the difference between the two divided by the average of the two, and it is presented as a $\%$.

| Description | Value | Uncertainty |  |
| :---: | :--- | :--- | :---: |
| Name of the product |  |  |  |
| Measured Diameter (cm) |  |  |  |
| Measured Height (cm) |  |  |  |
| Calculated Radius (cm) |  |  |  |
| Calculated Volume (cm $\left.{ }^{3}\right)$ |  |  |  |
| Density of the product (g/ml) |  |  |  |
| Searched value of density $(\mathbf{g} / \mathbf{m l})$ |  |  |  |
| \% Difference |  |  |  |

Table 2. Quantities measured and/or calculated in Part 2. Re-write the caption for the report such that it has complete information about the values from the table including description of how they were obtained (if measured, by what instrument? if calculated, by what formula?)

## Part 3: Constructing a box

You will need to construct a box of the same volume as the tin can. The base of the box must be a square with the side equals the diameter of the tin can.

1. Using formula (2) modified for the square base instead of circle, calculate the height of the box.
2. On any sheet of paper (you can recycle advertisements or wrappings), draw a blueprint of the box starting from the base square. There are different ways to add the rectangular sides to the base; examine boxes in your household and chose one you like.
3. Cut the pattern and assemble the box securing the sides with any tape, band aid, or glue strips.
4. Take a picture of the box with the can inside; explain why is there difference in height?
5. Include the picture in the report along with proper caption that contrast and compare the dimensions of the box and the tin can.

## Report:

1) The report must include an abstract of about one page summarizing the experience for all three parts of the lab. The abstract should not have any emotional descriptions such as "I was frustrated" or "I had fun doing the experiment". Points will be deducted for such irrelevant information or redundancy. The abstract must have a paragraph summarizing the outcomes of each part of the lab (Step 5 in Parts 1\&3, Step 7 in Part 2). Such paragraph should include important numerical values supporting the statements and reference the data tables containing the values.
2) Students have an option to submit an individual report but are encouraged to work with assigned lab partners. Only one report needs to be submitted per lab group from the Blackboard page (DO NOT email it to the lab instructor!). The names of the lab partners collaborating on the report must be listed in the header on each page of the report; a lab partner whose name is not listed will not receive the credit for the lab unless separate report is submitted individually (such report will not receive points for the collaboration).
3) Students can collaborate through WebEx (https://iol.tamucc.edu/webex.html), the Blackboard discussion board, or any other collaborative platforms such as Google Docs.
4) If the report is collaborative, the data presented in the report must consolidate the individual data from each lab partners and include as many versions of Tables 1 and 2 as names in the header but may have only one picture with 3 integrated images. The tables could be distinguished by sublabeling ( 1.1 or 1a, 1.2 or 1 b , etc.) with appropriate captions. However, only the best of three should be used (referenced in the abstract). Appendix might be created for the data tables that were NOT used in the abstract.
